Preliminary Estimates of Radiative Transfer Effects on Detached Shock Layers

ROBERT GOULARD*
General Motors Corporation, Santa Barbara, Calif

The effect of radiation losses on the structure of hypersonic detached shock layers is considered The formulation of this problem on the basis of a simple flow model illustrates the importance of the shock-layer optical thickness τ and that of the radiation convection ratio Γ Although an exact general solution requires numerical computations, several solutions are possible for small values of Γ through perturbation schemes Such solutions yield simple expressions for the temperature distribution, the heat flux, and the modified shock-layer thickness for several ranges of values of τ The limitations of these approximations and the difficulties inherent to their match with a viscous conducting boundary layer are discussed

Nomenclature

a	= velocity gradient [Eq (2)]
$A_L(a), A_L^+(a)$, $A_L^-(a)$	= see Eq (61)
B	= averaged Planck function ($\sigma T^4/\pi$)
c_p	= specific heat at constant pressure
E_n	= integro exponential functions (n = 1,2,3); see Ref 12, p 266
F_i^R	= radiation flux vector
h	= enthalpy
h_t	= total enthalpy $h + (u_i u_i/2)$
$H_a(x), H_a^-(x)$, $H_a^+(a)$	= see Eq (60)
k_R	= radiation volumetric absorption coefficient ($= \rho\kappa$)
k	= heat conductivity coefficient
n	= see Eq (22c)
N_R	= radiation-conduction parameter [Eq (48)]
p	= pressure
q^R	= net radiation heat flux normal to the wall
r	= distance of point M from axis of symmetry
R	= nose radius
$\mathcal{R}$	= perfect gas constant for air
t	= time
T	= absolute temperature
u,v	= velocity vector components
u_i	= velocity vector
U,V	= velocity vector components for the radiationless case
$dV(P)$	= elementary volume around point P
x,z	= coordinates of point M (Fig 1)
X,Z	= coordinates of point $M(x,z)$ for the radiationless case
β	= see Eq (22c)
Γ	= radiation convection parameter [Eq (4)]
δ	= shock-layer thickness
δ^*	= radiation boundary-layer thickness [Eq (50)]
Δ	= shock-layer thickness for the radiationless case
η	= optical thickness between point M and the wall for the radiationless case [Eq (55)]
θ	= see Fig 1
κ	= mass absorption coefficient averaged over the frequency spectrum
ρ	= density
$\tau_{MP}{}^S$	= optical length between M and P: $\tau_{MP}{}^S \equiv \int_M^P \rho\kappa ds$

Identical notation

u_i	= vector of components u_1, u_2, u_3
$S_{,i}$	= gradient of scalar S
$F_{i,i}$	= divergence of vector F_i
$u_i v_i$	= scalar product of vectors u_i and v_i

Subscripts

s	= immediately behind the shock
i	= indicial notation
0	= radiationless (reference) case
w	= conditions at the wall
∞	= upstream of shock

Superscripts

$\bar{\rho}, \bar{\mu}$,	= nondimensional quantities [Eqs (4) and (30)]

Presented as Preprint 63-452 at the AIAA Conference on Physics of Entry into Planetary Atmospheres, Cambridge, Mass, August 26–28, 1963; revision received January 6, 1964 It is a pleasure to acknowledge the fruitful discussions which took place while preparing this report with M M Baldwin, Eshenroeder, Heaslet, Vincenti, and Yoshikawa

* Consultant, Defense Research Laboratories; also Professor, School of Aeronautical and Engineering Sciences, Purdue University, Lafayette, Ind

Introduction

ATMOSPHERIC penetration speeds larger than escape velocity (36,000 fps) are becoming a matter of interest in various aspects of the space program [1,2] The desirability of short mission times and the interest in meteor studies are now creating a need for an understanding of fluid flow behavior at very high speed, where radiation becomes an important energy transfer contributor

The purpose of this paper is to study, on the basis of a simple flow model, the main aspects of the radiation energy transfer process in hypersonic detached shock layers The role of the radiation gas dynamics dimensionless parameters will be emphasized, and some approximate solutions will be given in closed form Although frequency dependence is an important consideration, a gray gas model will be assumed here to bring out more clearly the fluid dynamics aspects of the problem

I Stagnation Region of a Radiating Shock Layer

As in any fluid dynamics problem, the theoretical determination of the flow in the stagnation region of a hypersonic vehicle requires 1) the solution of the equations of conservation of mass, momentum, and energy in the fluid; and 2) adequate boundary conditions at the surface of the nose and at the shock interface

The conservation equations can be written for an ideal radiating gas in steady flow:

Mass

$$u_i(\partial\rho/\partial x_i) + \rho u_{i,i} = 0 \tag{1a}$$

Momentum

$$\rho u_i(\partial u_i/\partial x_i) = -p_{,i} \quad (1b)$$

Energy

$$\rho u_i(\partial h_t/\partial x_i) = -F_{i,i}^R \quad (1c)$$

where the viscous and heat-conduction terms have been neglected for simplicity For a cold absorbing wall [Ref 3, Eq (28)], the radiation loss term in Eq (1c) can be written as

$$F_{i,i}^R = \pi k_R(M)\left[4B(M) - \int_V k_R(P)B(P)\,\frac{\exp(-\tau_{MP}^S)}{\pi \overline{MP}^2}\,dV(P)\right] \quad (1d)$$

where the two terms on the right-hand side express, respectively, the energy radiated per unit time by the particles within a unit volume at the point M (Fig 1) and the energy received per unit time by these particles from all other particles at points P within the volume V The nondimensional term τ_{MP}^S corresponds to the optical length between M and P:

$$\tau_{MP}^S \equiv \int_{S=0}^{S=MP} k_R(s)ds$$

The boundary conditions are 1) at the wall, $u_i n_i = 0$ (n_i being the normal to the wall) and 2) at the shock interface, the Rankine-Hugoniot relations

Therefore, the determination of the steady inviscid radiant layer in the stagnation area involves, in general, the solution of a system of coupled integrodifferential equations in terms of two independent variables x and z (Fig 1)

A A One-Dimensional Model of the Nonradiating Shock Layer

The difficulty inherent to the manipulation of integral or differential equations of a two-dimensional character has inspired the applied mathematicians with a number of transformations and simplifications which generally tend to reduce the problem to a one-dimensional form

In some cases, such as the nonradiant shock layer, such simplification cannot be carried out rigorously, and available numerical solutions[4–6] represent rather each property in terms of both r and z (Fig 1) These results, however,[4 7] show that in the stagnation region there exists, for all practical purposes, no dependence of the flow properties on the distance r from the axis but only a dependence on the distance z from the wall † Also, for a wide range of hypersonic conditions the following hold:

1) The shock-layer thickness Δ is much smaller at very high speed than the radius R of the body ($\Delta/R \simeq \frac{1}{10}$) The stagnation area of the shock layer can then be assimilated to a one-dimensional gas slab

2) The density ρ and the thermal properties (T,h) are nearly constant in the stagnation area Also, the enthalpy far exceeds the kinetic energy: $h \gg (U^2 + V^2)/2$, hence $h_t \simeq h$

3) The velocity component V parallel to the axis of the body is nearly proportional to the distance from the surface of the body:

$$V \simeq -2aZ \simeq -(\rho_\infty V_\infty/\rho\,\Delta)Z \quad (2)$$

where the value substituted for a in the right-hand term satisfies the mass conservation equation at the shock

B Radiating Shock-Layer Problem

We now consider the case of the radiating layer ($F_{i,i}^R \neq 0$) The numerical iterations used in the nonradiant case[4–6] must now be modified to include the flux term $F_{i,i}^R$ in Eqs (1) In this case, one cannot expect the thermal properties to remain constant throughout the layer if a large part of the stored energy is drained by radiation Also, it is not possible to be sure (because of the long-range nature of radiation transfer) that introducing the radiation flux will not upset the one-dimensional character of the numerical nonradiant solution

Exact numerical solutions do not yet clarify this point, but it is possible first to seek out those cases where the radiation losses are small enough, in comparison with the stored energy associated with the flow, to be treated as a perturbation to the radiationless flow field

If one chooses the radiationless stagnation case as a reference state, it is possible to express Eq (1c) in nondimensional form [Ref 3, Eq (22c)]:

$$\bar{\rho}\bar{u}_i\bar{h}_{,i} = -\Gamma \bar{F}_{i,i}^R \quad (3)$$

where

$$\bar{\rho} \equiv \rho/\rho \qquad \bar{u} \equiv u/u \qquad \bar{h} \equiv h/h_s$$
$$\bar{x}_i \equiv x_i/\Delta \qquad \bar{F}_i^e \equiv F_i^R/q_0^R \qquad \Gamma \equiv q_0^R/\rho_\infty V_\infty h_s \quad (4)$$

The physical meaning of Γ in Fig 2 is the ratio of the energy q_0^R radiated per unit time and shock area by a shock layer in the reference state [i e , assuming that its properties $(\rho_0, T_0, \Delta,\ )$ are those calculated in the radiationless case] over the energy stored in the fluid crossing the shock, also per unit time and shock area

Equation (3) shows that radiation losses can be considered a perturbation to the state of shock layer when the value of Γ is small ‡ The radiating shock-layer problem, although complex in its general form, is therefore amenable, for the case where $\Gamma \ll 1$, to a one-dimensional perturbation solution; in this case, the simple radiationless shock-layer model will be used, modified to allow for small perturbations to the thermal properties and to the velocity

The interest of this approach is that it indicates with good accuracy the threshold of the radiation-gas dynamics cou-

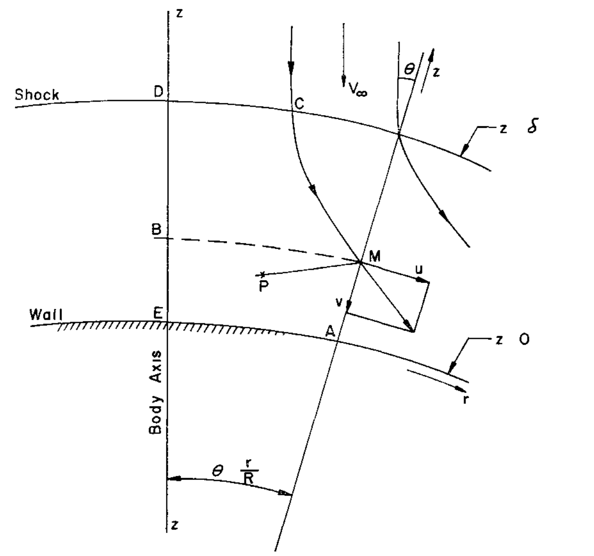

Fig 1 Detached shock-layer nomenclature

† This behavior naturally reminds us that such one dimensional behavior is obtained exactly in both potential flow and boundary-layer theory for an incompressible stagnation flow

‡ As pointed out by Eschenroeder,[32] one must make sure that Γ is truly representative of the flow configuration at hand In particular, when the only layers affected by radiation losses are those close to the wall (optically thick flows), the relevant value for the convected energy in Γ is the energy of these layers only, a small fraction of the total convected energy through the shock $\rho_\infty V_\infty h$ In this case the flow must be treated as a regular boundary-layer problem, as will be discussed in Sec III of this paper

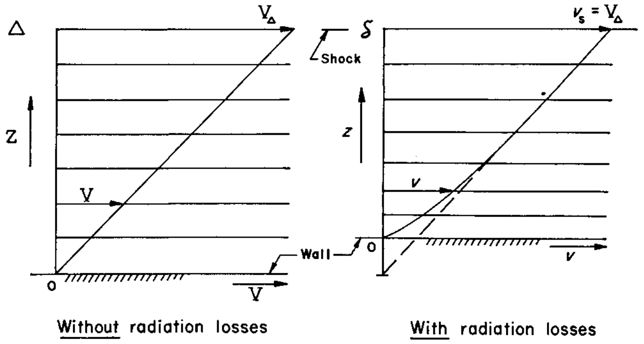

Fig 2 Vertical component v of the velocity

pling, that is, the range of flight regimes when radiation losses are beginning to affect the flow pattern

C A One-Dimensional Model of the Radiating Shock Layer

The analysis of the radiating shock layer can be carried out much in the same way as in radiationless shock-layer treatments, especially those also involving an energy sink term such as a chemical reaction term [8] Such an approach was recently used in the case of an optically thin gas [9] However, these studies, although instructive, lead to solutions of numerical nature and are therefore difficult to generalize Since our intention is to limit ourselves to small perturbations of the incompressible nonradiating model, we can further simplify our approach

Consider on Fig 1 the streamtube of axis $z'z''$ and of upstream radius DC The flow inside this tube passes successively through the circles of radius DC and BM and through the walls of the cylinder of axis $z'z''$, height BE or MA, and radius EA

Conservation of mass requires then that

$$\rho_\infty V_\infty \pi r_c^2 = \rho v \pi r^2 = 2\pi r \int_0^z \rho u dz \tag{5}$$

For the purpose of comparison with the radiationless case we note that, in this latter case ($\rho = \rho$), the same equations can be written

$$\rho_\infty V_\infty \pi r_c^2 = \rho \, V \pi r^2 = 2\pi r \rho \, UZ \tag{6}$$

where capital letters designate the radiationless ($\rho = \rho \simeq$ const) solution (where $\partial U/\partial Z \simeq 0$): u, v, and z take the values U, V, and Z, respectively

From these two sets of equations, a simple correspondence can be established between radiating and nonradiating shock layers:

$$\rho v(z) = \rho \, V(Z) \qquad \text{for a given } r \tag{7}$$

$$\int_0^z \rho u dz = \rho \, UZ \tag{8}$$

Turning now to the equation of state $p = \rho \mathcal{R} T$, we use for the pressure its Newtonian value, excluding the centrifugal force contribution since we are near stagnation,

$$p = \rho_\infty V_\infty^2 \cos^2\theta \simeq \rho_\infty V_\infty^2 [1 - (\theta^2/2)] \quad \text{with } \theta = r/R \tag{9}$$

Therefore, the pressure depends in first approximation on r only, and the assumption that ρ is inversely proportional to T along a given radial line (r = const) follows directly from the equation of state ($p = \rho \mathcal{R} T$):

$$\rho/\rho = T/T \tag{10}$$

Finally, the conservation of momentum along the direction r

$$\rho u \frac{\partial u}{\partial r} + \rho v \frac{\partial u}{\partial z} = - \frac{\partial p}{\partial r}$$

can be reduced [$\partial u/\partial z \ll \partial u/\partial r$, $\partial p/\partial r = -\rho_\infty V_\infty^2 (r/R^2)$] to the form

$$\rho u \frac{\partial u}{\partial r} = - \frac{\partial p}{\partial r}$$

and since [Eq (9)]

$$\frac{\partial p}{\partial r} \simeq - \rho_\infty V_\infty^2 \frac{r}{R^2}$$

we can write, in general,

$$\rho u du = -\rho_\infty V_\infty^2 (r/R^2) dr \qquad z = \text{const} \tag{11a}$$

and for the incompressible case,

$$\rho U dU = \rho_\infty V_\infty^2 (r/R^2) dr \tag{11b}$$

Since ρ is independent of x in the first approximation, Eqs (11a) and (11b) are easily integrated:

$$U = (\rho_\infty/\rho)^{1/2} (V_\infty/R) r \tag{12}$$

$$\rho^{1/2} u = \rho^{1/2} U \tag{13}$$

We note that the incompressible result [Eq (12)] is compatible with the radiationless model: $U = ar$ provided the constant a takes the value

$$a = (\rho_\infty/\rho)^{1/2} V_\infty/R \tag{14}$$

Comparing Eq (14) with the expression of the velocity constant a chosen for V [Eq (2)], we obtain an estimate of the incompressible layer thickness Δ of the same form as Probstein's early estimates[10] for a nearly flat nose: $\Delta \propto (\rho_s/\rho_\infty)^{1/2}$

The more interesting result, however, lies with Eq (13), which gives a means to eliminate the velocity u from Eq (8) After substitution, Eq (8) becomes

$$\rho^{1/2} dz = \rho^{1/2} dZ \tag{15}$$

which gives a convenient way to go, for a given value of r, from the incompressible coordinate Z associated with a streamline, to its actual physical coordinate z In particular, the actual shock-layer thickness δ is related to the radiationless shock-layer thickness Δ by the expression

$$\frac{\delta}{\Delta} = \frac{1}{\Delta} \int_0^\Delta \left(\frac{\rho_s}{\rho}\right)^{1/2} dZ \tag{16}$$

In summary, a radiating compressible shock layer can be related to the incompressible shock layer obtained for the same flow conditions but without radiation The flow model thus adopted for the study of radiation effects presents the following features:

1) Since radiation cooling will, if anything, reduce the shock-layer thickness, the assimilation of the shock layer to a one-dimensional slab will be maintained

2) The density ρ and the thermal properties (T,h,κ) do not vary with respect to r, but only as functions of z

3) The velocity component v parallel to the axis of the body is simply related to the velocity component V in the incompressible case by the expression

$$\rho v(z) = \rho \, V(Z) \tag{17}$$

4) The physical coordinate z is related to the coordinate Z of the corresponding radiationless case by the expression

$$\rho^{1/2} dz = \rho^{1/2} dZ \tag{18}$$

5) The density variations across the shock layer are related to the temperature variations by the expression

$$\rho/\rho \simeq T/T \tag{19}$$

6) The conditions at the shock of both compressible and incompressible shock layers are identical for a given flight condition (Rankine-Hugoniot)

D Radiating Shock Layer for $\Gamma \ll 1$

In this one-dimensional model ($\partial/\partial r = 0$, and $h \simeq h_0$), Eq (1c) can be reduced to

$$\rho v \frac{\partial h}{\partial z} = -\frac{\partial q^R}{\partial z} \tag{20}$$

where the radiation flux term can be reduced [Ref 3, Eq (31)] to

$$\frac{\partial q^R}{\partial z} = \pi k_R [4B(\tau) - 2\int_0^{\tau} B(t)E_1(\tau - t)dt - 2\int_{\tau}^{\tau_\delta} B(t)E_1(t - \tau)dt] \tag{21}$$

where the integro-exponential functions E_1 correspond to the integıation over angles of the right-hand side second term of Eq (1d) for the one-dimensional case This function is tabulated, for instance, in Ref 12, p 266 The term τ is the optical thickness $d\tau = k_R dz$

If we further consider that in the chosen model

$$dz = (\rho\ /\rho)^{1/2} dZ \tag{22a}$$

$$\rho v = \rho\ V = -2\rho\ aV \tag{22b}$$

with

$$2a = \rho_\infty V_\infty / \rho\ \Delta$$

and

$$k_R = \rho\kappa \tag{22c}$$

with $\kappa/\kappa\ \simeq (\rho/\rho\ )^n (T/T\ )^\beta$, the convection term can be written

$$\rho v c_p \frac{\partial T}{\partial z} = -\frac{\rho_\infty V_\infty}{\Delta}\left(\frac{\rho}{\rho}\right)^{1/2} c_p Z \frac{\partial T}{\partial Z}$$

and Eq (5) becomes

$$-\frac{\rho_\infty V_\infty}{\Delta}\left(\frac{\rho}{\rho}\right)^{1/2} c_p Z \frac{\partial T}{\partial Z} = \rho\kappa[4\sigma T^4 - 2\int_0^{\tau} T^4(t)E_1(\tau - t)dt - 2\int_{\tau}^{\tau_\delta} T^4(t)E_1(t - \tau)dt] \tag{23}$$

Equation (23) is a ıelation between Z and T, or functions of T, since the properties ρ, κ, and c_p can be expressed in terms of T Therefore, it is seen that, on the basis of a somewhat crude but essentially correct flow model of the stagnation shock layer, the determination of the radiation properties of this layer is reduced to the solution of a single equation $T(Z)$

A simple Doıodnitzin-like transformation [Eq (18)] expresses this solution in terms of the physical coordinate z (see Fig 2)

The purpose of this article is to exploıe the various foıms of this solution foı different radiation ıegimes

E Asymptotic Solutions foı $\Gamma \to 0$

Similarity analysis[3] shows that, in addition to the parameter $\Gamma = q_0^R/\rho_\infty V_\infty h$, the optical thickness $\tau_\delta = k_R\delta$ of the shock layer governs the mathematical form of Eq (23) and of its solution

In the limit case $\Gamma \to 0$, the ıadiation from the isothermal shock layer to the wall $q_{0\ w}^R$ or through the shock $q_0\ ^R$ [Ref 11, Eq (25)] is

$$q_{0\ w}^R = q_0\ ^R = \sigma T^4[1 - 2E_3(\tau_\Delta)] \tag{24}$$

where the function $E_3(x)$ is tabulated in a number of sources (e g , Ref 12, p 266) Two limit cases for $\tau_\Delta \ll 1$ aıe

$$E_3(\tau_\Delta) \to \tfrac{1}{2} - \tau_\Delta \text{ and } q_{0\ w}^R = q_{0\ s}^R = 2\sigma T^4 \tau^\Delta \tag{25}$$

and for $\tau_\Delta \gg 1$,

$$E_3(\tau_\Delta) \simeq \frac{e^{-\tau_\Delta}}{\tau_\Delta} \to 0 \text{ and } q_{0\ w}^R = q_0\ ^R = \sigma T^4 \tag{26}$$

In the more general case $\Gamma \ll 1$, Eq (23) can also take two asymptotic forms when τ_Δ is very small or very large These two cases, as well as the general case $\tau_\Delta \simeq 1$, will be discussed in the next three sections The condition $\Gamma \ll 1$ allows us to use the perturbation assumptions built in the shock model

II Optically Thin Layer ($\tau_\Delta \ll 1$)

In this case, the integral terms on the right-hand side of Eq (23) can be shown[3] to be negligible in comparison with the term $4\sigma k_R T^4$ (no self-absorption) Equation (23) reduces to

$$-\frac{\rho_\infty V_\infty}{\Delta}\left(\frac{\rho}{\rho}\right)^{1/2} c_p Z \frac{\partial T}{\partial Z} = 4\rho\kappa\sigma T^4 \tag{27}$$

which can be rearranged into

$$\frac{dZ}{Z} = \frac{\rho_\infty V_\infty h_s}{4\rho\ \kappa\ \Delta\sigma T\ ^4}\left(\frac{\rho}{\rho}\right)^{-1/2}\frac{\kappa}{\kappa}\left(\frac{T_s}{T}\right)^4\frac{c_p dT}{h} \tag{28}$$

Introducing now, in addition to the radiation-convection ıatio Γ_n for optically thin§ gases [Eq (4)] {where the total radiation loss flux is $q_0^R = q_{0\ w}^R + q_0\ ^R$ [Eq (25)]},

$$\Gamma_n \equiv \frac{4\rho_s\kappa_s\Delta\sigma T\ ^4}{\rho_\infty V_\infty h} \tag{29}$$

the nondimensional teıms

$$\bar{T} = \frac{T}{T} \qquad \bar{Z} = \frac{Z}{\Delta} \qquad \bar{\rho} = \frac{\rho}{\rho} \simeq \frac{T_s}{T} = \bar{T}^{-1}$$

$$\frac{c_p}{(h\ /T\ )} = \left(\frac{T}{T}\right)^\alpha \qquad \frac{\kappa}{\kappa_s} \simeq \left(\frac{\rho}{\rho}\right)^n\left(\frac{T}{T}\right)^\beta = \bar{T}^{\beta - n} \tag{30}$$

wheıe the power dependence of the thermodynamic functions c_p, κ is justified for the case of small perturbation from the radiationless case, we obtain

$$d\bar{Z}/\bar{Z} = \Gamma_n^{-1}\bar{T}^{\alpha+\beta-n+(1/2)-4}d\bar{T} \tag{31}$$

The solution of this equation yields

$$\bar{T} = [1 + (\alpha + \beta - n + \tfrac{1}{2} - 3)\Gamma_n \ln\bar{Z}]^{1/[\alpha+\beta-n+(1/2)-3]} \tag{32}$$

Foı low values of Γ_n, this expression may be expanded thıough the binomial theorem, and we obtain

$$\bar{T} = 1 + \Gamma_n \ln\bar{Z} \tag{33}$$

To find the total heat flux to the wall (also equal to the outwaıd flux thıough the shock for $\tau_\Delta \ll 1$), it is enough to sum all contributions:

$$q_w^R = \int_0^\delta 2\rho\kappa\sigma T^4 dz = 2\rho\ \kappa_s\sigma T\ ^4\int_0^\delta \bar{T}^{\beta-n+3}dz \tag{34}$$

and, intıoducing the substitution of variables $dz = (\bar{\rho})^{-1/2} dZ = \bar{T}^{1/2}dZ$ [Eqs (18) and (30)], and the ratio $\bar{Z} = Z/\Delta$,

$$q_w^R = 2\rho\ \kappa\ \Delta\sigma T\ ^4\int_0^1 \bar{T}^{\beta-n+3+(1/2)}d\bar{Z} \tag{35}$$

wheıe $2\rho\ \kappa\ \Delta\sigma T\ ^4$ is the zero-order solution $q_{0\ w}^R$ [Eq (25)] After substitution of Eqs (25) into Eq (35) we obtain the

§ The notation Γ_n and Γ_κ for the radiation convection ratio is used here for the optically thin and thick gases, respectively; it is thus easier to identify than Unsöld's original Γ' and Γ''

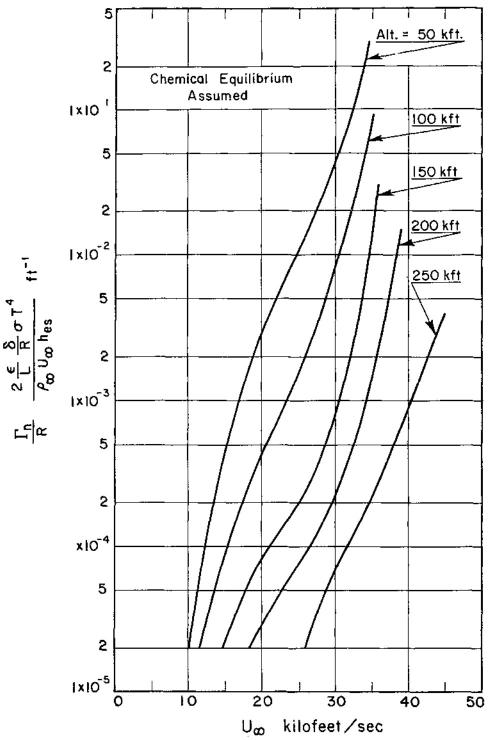

Fig 3 Radiation-convection ratio vs velocity for different altitudes assuming chemical equilibrium (from Ref 7)

reduced flux

$$\bar{q}_w^R \equiv \frac{q_w^R}{q_{0,w}^R} = \int_0^1 \bar{T}^{\beta-n+3+(1/2)} d\bar{Z} \tag{36}$$

and now using Eq (33) and expanding $\bar{T}^{\beta-n+3-(1/2)}$ through the binomial theorem,

$$q_w^R \cong \int_0^1 [1 + (\beta + 3 - n - \tfrac{1}{2})\Gamma \ \ln\bar{Z}]d\bar{Z} = 1 - (\beta - n + 3 - \tfrac{1}{2})\Gamma_n \tag{37}$$

Finally, the variation of shock-layer thickness from its radiationless value Δ (see Fig 3) can be easily established from Eq (16):

$$\frac{\delta}{\Delta} = \int_0^1 \bar{\rho}^{-1/2} d\bar{Z} = \int_0^1 \bar{T}^{1/2} d\bar{Z} \tag{38}$$

After substitution of Eqs (33) and (16a), we obtain

$$\frac{\delta}{\Delta} \simeq \int_0^1 (1 + \Gamma_n \ln\bar{Z})^{1/2} d\bar{Z} \tag{39}$$

Expanding once more through the binomial theorem ($\Gamma_n \ll 1$),

$$\frac{\delta}{\Delta} = 1 - \tfrac{1}{2}\Gamma_n\left[\lim_{\bar{z}\to 0}\left(\int_{\bar{z}}^1 \ln\bar{Z} d\bar{Z}\right)\right] = 1 - \tfrac{1}{2}\Gamma_n \tag{40}$$

In summary,¶ for an optically thin gas ($\tau_\delta \ll 1$) and $\Gamma \ll 1$,

$$\bar{T} = 1 + \Gamma_n \ln\bar{Z}$$

$$\bar{q}_w^R = \bar{q}^R = 1 - (\beta + 3 - n + \tfrac{1}{2})\Gamma_n \tag{41}$$

$$\bar{\delta} = 1 - \tfrac{1}{2}\Gamma_n$$

¶ Note: Since Δ and, consequently, both $q_{0,w}^R$ and Γ_n are proportional to the nose radius R, Eq (41) shows that q_w^R and δ are quadratic functions of R This simple relationship seems to be verified by the numerical results of Howe (Figs 14 and 17 of Ref 13)

with

$$\bar{z} = \int_0^z \bar{T}^{1/2} dZ = Z + \frac{\Delta\Gamma_n}{2}\int_0^{\bar{z}} \ln\bar{Z} d\bar{Z}$$

$$\Gamma_n = \frac{4\rho\,\kappa_s \Delta\sigma T_s^4}{\rho_\infty V_\infty h}$$

Various values of n and β have been suggested for high-temperature air Recent estimates (see, for instance, Fig 3 of Ref 14) show that $n \simeq 0$ and β increases from about 4 at 5000°K to a maximum of about 8 in the 12,000°K range and finally drops to lower values beyond 15,000°K

If we chose $\beta = 7$ as a representative value for the range where radiation losses become important [$T > 10{,}000$°K],[7] the expression of the reduced flux in Eq (41) becomes

$$\bar{q}_w^R = 1 - (\beta + 3)\Gamma_n \simeq 1 - 10\Gamma_n \tag{42}$$

Consequently, whenever Γ_n is larger than $\frac{1}{100}$, derivations from the isothermal estimates of the flux $q_{0,w}^R$ are beginning to exceed 10% Figure 3, extracted from Ref 7, gives the flight ranges corresponding to different values of Γ_n

Also, the continuously decreasing temperature of the particles as they travel from the shock to the wall [Eq (41)] leads to cooler boundary-layer edge conditions than for the radiationless case: hence, a conduction heat-transfer reduction from its classical radiationless formulation Some gross estimates of this coupling were made in Ref 7

It should also be noted at this point that even within the framework of a totally inviscid shock layer, Eq (33) shows that, for any given value of Γ_n, the small perturbation hypothesis will tend to lose its validity when Z tends to zero, since then the absolute value of the product $\Gamma_n \ln\bar{Z}$ eventually becomes comparable to unity As a consequence, the lower limit of the integrals shown in Eqs (37) and (39) should be a small quantity rather than zero Hence, the expressions for $\bar{q}^R$ and $\bar{\delta}$ in Eqs (41) are slightly overestimating the cooling effect; they are a lower bound to the actual value of $\bar{q}^R$ and δ, tending to be more exact when Γ_n tends to zero

An additional remark is that the boundary layer at the wall cuts into those flow layers mostly affected by the radiation cooling effect, i e , those layers nearest to the wall Since viscous and radiative phenomena can hardly be considered additive, a careful study would be necessary to account for this simultaneous transfer by radiation and conduction The inviscid solution offered here will tend to be correct when the boundary-layer thickness is small compared to the shock-layer thickness; this will be true mainly at low altitude and for turbulent boundary layers At higher altitudes, neither the inviscid approximation nor the chemical equilibrium assumptions used in this paper are satisfactory, and more work is needed in this area [13–16 33]

III Optically Thick Layer ($\tau_\Delta \gg 1$)

In optically thick shock layers, radiation travels only short distances, and the energy losses to the outside of the layer will tend to occur in two boundary layers only: near the shock and near the wall Therefore, the perturbation scheme used in Sec II when the losses were distributed throughout the shock layer must now be replaced by a boundary-layer approach, where large losses and property variations are allowed to take place within thin layers On the sketch shown on Fig 4, one can recognize the three areas crossed in succession by a particle entering the shock layer: the shock boundary layer (AB), the isothermal shock layer (BC), and the wall boundary layer (CD)

The following estimates can be made on these three regions

A Shock Boundary Layer (AB)

The initial part of the shock layer can be reasonably likened to the one-dimensional normal shock problem since the "shock boundary layer" is thin and possesses in first approximation a constant mass flux along $z'z''(\rho v = \rho_\infty V_\infty)$

In this case, recent work on "radiation resisted shock waves"[17, 18] describes fairly well this part of the flow If one further assumes the usual shock discontinuity due to the neglect of viscosity and conduction, plus zero absorption in the cold upstream gas, the problem is reduced[19] to the effect of the radiation losses on a gas heated at the shock discontinuity ($T = T$) according to the radiationless Rankine-Hugoniot conditions Such losses tend to reduce, after a few optical lengths, the temperature behind the shock to an asymptotic value T_2 [see Fig 11 of Ref 19]

B Isothermal Shock Layer (*BC*)

This part of the shock layer, which tends to be its major portion if the shock layer is optically thick enough, is therefore determined by the history of the previous layer *AB* One notes that the forward radiation flux

$$q^R = 2\int_0^\infty \sigma T^4(\tau) E_2(\tau) d\tau \tag{43}$$

is larger than the blackbody value

$$q_{bb}^R = \sigma T_2^4 \tag{44}$$

since this blackbody value would be the value given by Eq (43) if T were always equal to T_2, whereas it is actually greater near the shock (AB)

To estimate simply (as in Refs 20 and 1) the conditions in the layer BC (subscript 2) by satisfying the energy equation

$$h_\infty + \frac{V_\infty^2}{2} = h_2 + \frac{V_2^2}{2} - \frac{\sigma T_2^4}{\rho_\infty V_\infty} \tag{45}$$

introduces, therefore, an underestimate of the radiation losses,** and the temperature T_2 obtained in Ref 21 will be an upper limit of the real value Conversely, the values shown in Ref 21 for the density ratio across the shock are a lower limit of the actual density ratio ρ_2/ρ_∞ Even so, they turn out to be much larger than for the radiationless case The optically thick shock layer is therefore expected to be physically thinner than the radiationless layer †† Equation (45) shows clearly that the governing parameter for this variation of δ is the Boltzmann number:

$$Bo \equiv \frac{\rho_\infty V_\infty [h_\infty + (V_\infty^2/2)]}{\sigma T_2^4} \tag{46}$$

C Wall Boundary Layer

Near the wall, the right-hand side of Eq (23) takes a differential form first suggested by Rosseland[23]:

$$q^R = -\frac{16\sigma T^3}{3k_R}\frac{\partial T}{\partial z} \tag{47}$$

Since we are concerned in this case ($\Gamma \ll 1$, $\tau_\delta \gg 1$) with the layers near the wall, it is appropriate to examine more closely the assumption that conduction could be uncoupled from the radiation problem In Eq (23), it amounts to neglecting the conduction term $q^c = -k_c(\partial T/\partial z)$ in comparison with the radiation term q^R [Eq (47)] In other words, the following assumption is made:

$$N_{R-c} \equiv \frac{q^R}{q^c} = \frac{16\sigma T^3}{3k_c k_R} \gg 1 \tag{48}$$

** The writer is indebted to Baldwin and Heaslet for pointing out this discrepancy in Refs 20 and 21 This situation parallels that of the blackbody-Rosseland discrepancy at the wall [22]

†† This effect tends to be attenuated when the cold air ahead of the shock absorbs an appreciable fraction of the forward radiation flux; this energy is convected back into the shock layer, and Eq (45) must be modified

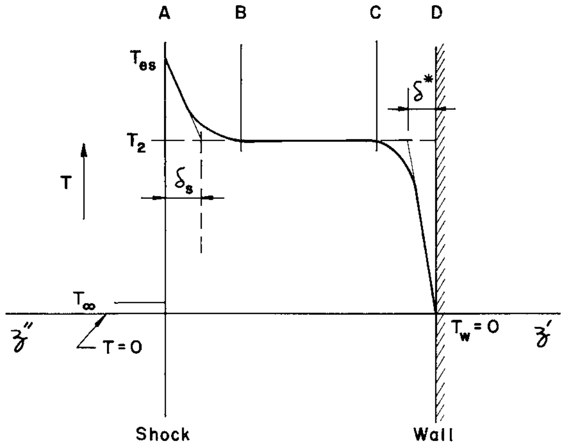

Fig 4 Typical temperature profile of an optically thick shock layer

The influence of this parameter, quite akin to Sen and Guess' "radiation broadening factor g,"[24] has been recently investigated by Viskanta and Grosh[25] for wedge flows For the important case $N_{R-c} \gg 1$, assuming $k_R \propto T^3$, Eq (47) yields a simple closed solution[22] of the following form‡‡:

$$q^R = \frac{16\sigma T_2^4}{3k_{R2}}\frac{1}{\delta^*} \tag{49}$$

with $\delta^* \equiv$ "radiation-layer thickness"

$$\delta^* = \Delta(2\pi\Gamma)^{1/2} \tag{50}$$

$$\Gamma \equiv \frac{q_0^R}{\rho_\infty V_\infty h_s} = \frac{16\sigma T_2^4}{3\rho_\infty V_\infty h}\frac{1}{k_R \Delta} \tag{51}$$

Equation (51) defines a value of Γ_κ based arbitrarily on the shock-layer thickness Δ in the expression of a typical Rosseland radiation flux q_0^R:

$$q_0^R = \frac{16\sigma T_2^3}{3k_{Rs}}\frac{T_2}{\Delta} \tag{52}$$

Although Δ is the only length known a priori in this problem, Eq (49) shows that the meaningful length in this case is not the shock-layer thickness Δ but the radiation boundary-layer thickness δ^* The analogy with the conduction boundary-layer thickness is emphasized by the dependence of δ^* on $\Gamma_\kappa^{1/2}$, since Γ is the direct analog in radiation gas dynamics [Ref 3, Eq (15); Ref 26, Eq (12 6)] of the inverse of the Peclet number ($Pe \equiv Re \times Pr$)

Another asymptotic solution to this problem is when Γ_κ becomes so small as to obtain a very thin "radiation boundary layer" (small δ^*) where very high temperature gradients exclude the validity of Rosseland's formulation Since the shock layer tends, in this case, to become a completely isothermal slab, we can expect at the limit ($\Gamma_\kappa \to 0$) that the blackbody formulation $q_w^R = \sigma T^4$ is the correct form to use The transition from one form to another is quite analogous to a slip-flow regime and was discussed in Refs 22, 27, and 28

‡‡ Equation (19) of Ref 22 gives, on the basis of an arbitrary reference state (V,L), $\delta^* = L(\pi\Gamma_L)^{1/2}$, where $V = -La$ and

$$\Gamma_L = \frac{16\sigma T_i^3}{3Vc_p\rho}\frac{1}{k_i L}$$

In the model used here, let us choose $L = \Delta$; hence, $V = (\rho_\infty V_\infty/2\rho)$ and

$$\Gamma_L = \frac{16\sigma T_s^4}{\frac{3}{2}(\rho_s/\rho_\infty)V_\infty\rho_\infty h_s}\frac{1}{k_{Rs}\Delta}$$

Therefore $\Gamma_L = 2\Gamma_k$ as defined in Eq (51) and $\delta^* = \Delta(2\pi\Gamma_k)^{1/2}$

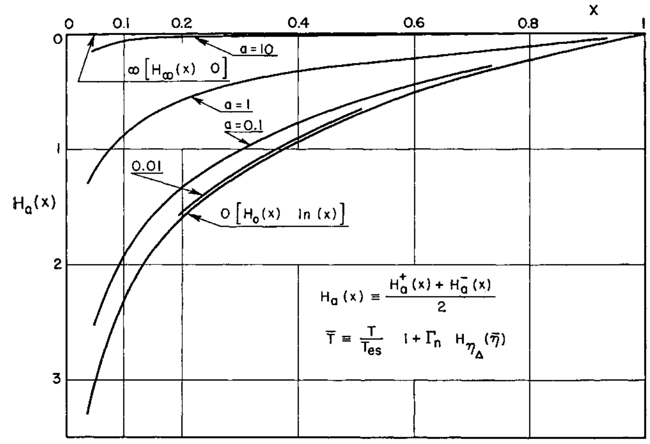

Fig 5 The integral $H_a(x)$ for various values of a and n

An interesting problem would be to establish the range of flight conditions for which BC reduces to zero, and the two thermally conducting layers AB and CD merge into one

IV Finite Optical Thickness

The radiation loss term on the right-hand side of Eq (23) must now be used fully:

$$-\frac{\rho_\infty V_\infty}{\Delta}\left(\frac{\rho}{\rho}\right)^{1/2} c_p Z \frac{\partial T}{\partial Z} = \rho\kappa[4\sigma T^4 - 2\int_0^\tau \sigma T^4(t)E_1(\tau - t)dt - 2\int_\tau^{\tau_\delta} \sigma T^4(t)E_1(t-\tau)dt] \quad (53)$$

The solution of this equation requires a numerical integration procedure A first approximation is often obtained by using the "local temperature approximation" In this method, each property under the integral signs is expanded in a Taylor series around its local value ($t = \tau$), and only the first term is conserved For instance,

$$\int_0^\tau T^4(t)E_1(\tau - t)dt = \int_0^\tau \left[T^4(\tau) + \frac{\partial T^4(\tau)}{\partial t}(\tau - t) + \quad\right] E_1(\tau - t)dt \simeq \int_0^\tau T^4(\tau)E_1(\tau - t)dt$$

Equation (53) can then be rewritten, after integration of

$$E_1 \left[\int_0^x E_1(x')dx' = 1 - E_2(x)\right]$$

as follows:

$$-\frac{\rho_\infty V_\infty}{\Delta}\left(\frac{\rho}{\rho_\infty}\right)^{1/2} c_p Z \frac{\partial T}{\partial Z} = 2\rho\kappa\sigma T^4[E_2(\tau) + E_2(\tau_\delta - \tau)] \quad (54)$$

Equation (54) contains two dependent variables Z and τ One must be eliminated By definition,

$$\tau \equiv \int_0^Z \rho\kappa dz$$

Upon introduction of the variable

$$\eta \equiv \rho\,\kappa_s Z \quad (55)$$

and using the transformation (z,Z) [Eq (15)] we can write in first approximation

$$\tau = \int_0^z \rho^{1/2}\kappa\rho^{1/2}dz = \rho^{\ 1/2}\int_0^Z \rho^{1/2}\kappa dZ = \rho\,\kappa\int_0^Z \bar{T}^{\beta+n+(1/2)}dZ \simeq \rho_e\,\kappa\,Z + \quad \simeq \eta$$

Hence the new form of Eq (54) in $T(\eta)$:

$$-\frac{\rho_\infty V_\infty}{\Delta}\left(\frac{\rho}{\rho}\right)^{1/2} c_p\eta \frac{\partial T}{\partial \eta} = 2\rho\kappa\sigma T^4[E_2(\eta) + E_2(\eta_\Delta - \eta)] \quad (56)$$

where η_Δ is the optical thickness of the "radiationless" shock layer:

$$\eta_\Delta = \rho\,\kappa\,\Delta$$

After separation of variables, Eq (56) becomes

$$\tfrac{1}{2}[E_2(\eta) + E_2(\eta_\Delta - \eta)]\frac{\partial\eta}{\eta} = \frac{\rho_\infty V_\infty h_s}{4\rho\,\kappa\,\Delta\sigma T^{\,4}}\left(\frac{\rho}{\rho}\right)^{-1/2}\frac{\kappa}{\kappa}\frac{T_s^{\,4}}{T^4}\frac{c_p dT}{h} \quad (57)$$

Introducing $\bar{\eta} \equiv \eta/\eta$ and the dimensionless variables of Eqs (29) and (30), Eq (57) reduces to the form ($\Gamma_n \ll 1$):

$$\frac{1}{2}\int_{\bar{\eta}}^1 \{E_2(\eta_\Delta\bar{\eta}) + E_2[\eta_\Delta(1-\bar{\eta})]\}\frac{d\eta}{\eta} = \Gamma_n^{-1}\bar{T}^{\alpha+\beta-n+(1/2)-4}d\bar{T} \quad (58)$$

A Approximate and Numerical Solutions

Clearly, Eq (58) reduces to Eq (31) when $\eta_\Delta \to 0[E_2(0) = 1]$

Before turning to numerical operations, let us consider Lunev's assumption[29] that the sum

$$\tfrac{1}{2}\{E_2(\eta_\Delta\bar{\eta}) + E_2[\eta_\Delta(1-\bar{\eta})]\}$$

is, on the average, equal to $E_2(\frac{1}{2}\eta_\Delta)$ In that case, this sum can be taken out of the integral on the left-hand side of Eq (58), which then becomes identical to Eq (31) provided Γ_n be replaced in (31) by the product $\Gamma_n E_2(\frac{1}{2}\eta_\Delta)$ Consequently, the results obtained in Sec II of this report for $\bar{T}$ and $\bar{\delta}$ in the optically thin case [Eq (41)] can be modified for the self-absorption case ($\eta_\Delta \lesssim 0\ 2$) by substituting the product $\Gamma_n E_2(\frac{1}{2}\eta_\Delta)$ to the radiation-convection ratio Γ_n The modification for the reduced heat flux is not as simple

One may further comment that the correct parameter Γ to be used in this self-absorbing case should have as a numerator [Eqs (24) and (25)] the quantity $q_0^R = \sigma T^4[1 - 2E_3(\eta_\Delta)]$ and not the optically thin value that appears in Γ_n, that is, $q_{0\ \mathrm{thin}}^R = 2\sigma T^4\eta_\Delta$ One can easily verify that, for $\eta_\Delta < 0\ 3$, the ratio

$$\frac{q_{0,\mathrm{thin}}^R E_2(\frac{1}{2}\eta_\Delta)}{q_0^R} = \frac{2E_2(\frac{1}{2}\eta_\Delta)\eta_\Delta}{1 - 2E_3(\eta_\Delta)} \simeq 1 \quad (59)$$

within 5%

Therefore, one can extend the validity of Eqs (41) for $\bar{\delta}$ and $\bar{T}$ to an optical thickness of 0 3 or so, provided Γ_n be replaced by Γ where the appropriate formulation of the radiative flux q_0^R is used at the numerator [Eq (24)]

When the gas has a substantial optical thickness ($\eta_\Delta >$ 0 3), it is necessary to calculate the integrals

$$H_a^+(x) \equiv \int_1^x E_2(a\xi)\frac{d\xi}{\xi}$$

$$H_a^-(x) \equiv \int_1^x E_2[a(1-\xi)]\frac{d\xi}{\xi} \quad (60)$$

$$H_a(x) \equiv [H_a^+(x) + H_a^-(x)]/2$$

These functions have been tabulated by Chen, and the function $H_a(x)$ is illustrated in Fig 5 for several values of a One notes in Fig 5 that a good approximation of $H_a(z)$ is the product $(\ln x)E_2(\frac{1}{2}\eta_\Delta)$ This allows us to extend Lunev's approximation[29] to optical thicknesses of order unity or more [see Eq (62)], although the further simplification shown on

Eq (59) cannot be used beyond $\eta_\Delta > 0\ 3$ The following functions are also useful:

$$A_L{}^+(a) \equiv \int_L^1 H_a{}^+(x)dx$$
$$A_L{}^-(a) \equiv \int_L^1 H_a{}^-(x)dx \qquad (61)$$
$$A_L(a) \equiv \frac{A_L{}^+(a) + A_L{}^-(a)}{2} = \int_L^1 H_a(x)dx$$

These functions have also been tabulated by Chen for various values of a and L, and $A_L(a)$

Integrating now, Eq (58) yields, after steps similar to those leading to Eq (33),

$$\bar{T} \simeq 1 + \Gamma_n H_{\eta_\Delta}(\bar{\eta}) \simeq 1 + [\Gamma_n E_2(\tfrac{1}{2}\eta_\Delta)] \ln\bar{\eta} \qquad (62)$$

Similarly, substitution of Eq (62) into Eq (39) yields, for the shock-layer thickness,

$$\frac{\delta}{\Delta} \simeq 1 - \tfrac{1}{2}\Gamma_n \left\{ \lim_{\bar{\eta}\to 0} \left[\int_{\bar{\eta}}^1 H_{\bar{\eta}_\Delta}(\eta')d\bar{\eta}' \right] \right\}$$

Hence,

$$\frac{\delta}{\Delta} \simeq 1 - \tfrac{1}{2}\Gamma_n A_0(\eta_\Delta) \simeq 1 - \tfrac{1}{2}\left[\Gamma_n E_2\left(\frac{\eta_\Delta}{2}\right)\right] \qquad (63)$$

Finally, the expressions for the radiation flux towards the wall $q_w{}^R$ and towards the shock $q\ {}^R$ are, within the present approximation,

$$q_w{}^R = \int_0^1 \sigma T^4 E_2(\eta_\Delta\bar{\eta})d\bar{\eta}$$
$$q_s{}^R = \int_0^1 \sigma T^4 E_2[\eta_\Delta(1-\bar{\eta})]d\bar{\eta} \qquad (64)$$

Substitution of the temperature T from Eq (62) gives a value for both fluxes in Eqs (64) Since the function $E_2(\eta_\Delta\bar{\eta})$ will be minimum and $E_2[\eta_\Delta(1-\bar{\eta})]$ will be maximum when T is itself maximum ($\bar{\eta} \to 1$), it is apparent that always $q\ {}^R \geqslant q_w{}^R$ This conclusion is borne out by the numerical results of Ref 19, and differs from the optically thin case where $q\ {}_n{}^R = q_w\ {}_n{}^R$ [Eq (37)]

B Results and Discussion

For a given Γ_n, it is clear from Eq (62) and Fig 5 that an increasing optical thickness means diminishing radiation losses from the internal layers, and the shock layer tends, as expected, to be isotheımal when η_Δ grows large (blackbody)

Similarly, the integral $A_0(\eta_\Delta)$, approximated by $E_2(\eta_\Delta/2)$, decreases from unity for $\eta_\Delta = 0$, to zero for large values of η_Δ Equation (63) shows, therefore, that for a given Γ_n the shock-layer thickness δ tends to its radiationless value Δ when η_Δ becomes large

To illustrate this result, let us consider a grossly simplified atmospheric model[28] where the density ratio across the shock wave and, therefore, the incompressible shock-layer nose-radius ratio $\rho\ /\rho_\infty$, are constant for all regimes Let us take also $\kappa \propto \rho^n T^\beta (n = \frac{1}{2}$ and $\beta = 7)$ and $T \propto u_\infty$

In these conditions (see Fig 6),

$$\Gamma_n = \frac{4\rho\ \kappa\ \sigma T\ ^4\Delta}{\frac{1}{2}\rho_\infty V_\infty{}^3} \propto \rho_\infty{}^n T\ ^{\beta+1}R \propto \rho_\infty{}^n u_\infty{}^{\beta+1}R$$

$$\eta_\Delta = \rho\kappa\ \Delta \propto \rho_\infty{}^{n+1}T\ ^\beta R \propto \rho_\infty{}^{n+1}u_\infty{}^\beta R$$

Now consider the following (Fig 7)

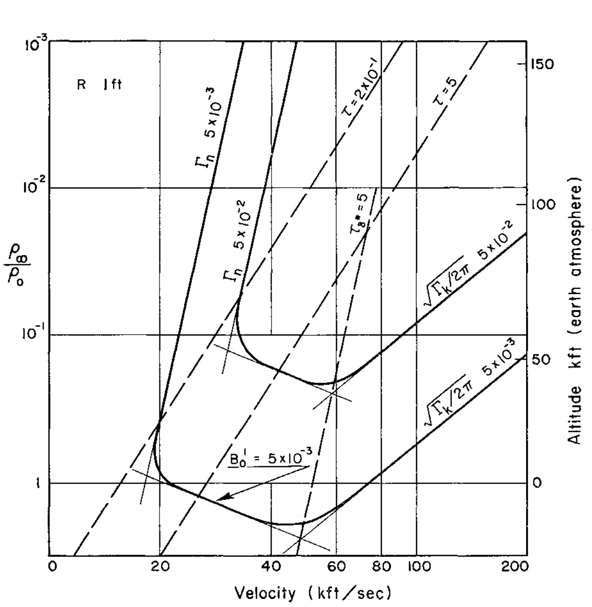

Fig 6 Radiation transfer regimes for detached shock layers in an ideal planetary atmospheıe (from Ref 28)

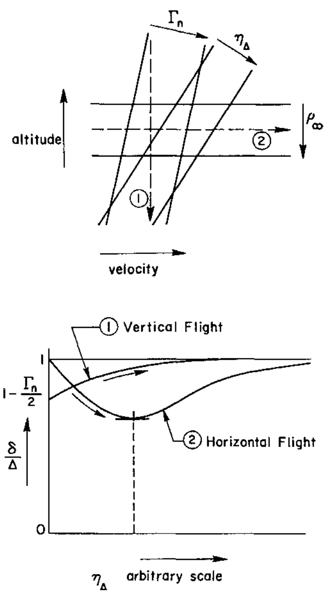

Fig 7 Radiation-induced variations of shock-layer thickness in two flight cases, 1 and 2

1 *A vertical entry* (u_∞ = *const*)

In this case, the object follows a line of $\Gamma_n \propto \rho^n$ since u_∞ = const On the other hand, the optical thickness η_Δ increases rapidly with $\rho_\infty{}^{n+1}$ Therefore, the function $A_0(n_\Delta)$ tends rapidly from unity to zero, and the shock-layer thickness, as it becomes optically thicker in the denser layers of the atmosphere, increases from its initial minimum value $\delta = \Delta(1 - \frac{1}{2}\Gamma_n)$ [see Eq (41)] to its self-absorbing value $\delta \simeq \Delta$ It seems that the maximum collapse of the shock layer due to radiation cooling should take place in the early part of the penetration

2 *A horizontal accelerated flight* (ρ_∞ = *const*)

In this case, Γ_n increases rapidly with velocity, as does the optical thickness η_Δ Since for ρ_∞ = const, Γ_n varies about linearly with η_Δ, it is interesting, in Eq (63), to follow the variations of the product

$$\eta_\Delta A_0(\eta_\Delta) \simeq \eta_\Delta E_2(\eta_\Delta/2)$$

Except near $\eta_\Delta = 0$, this product is well approximated by $\frac{2}{3}\eta_\Delta e^{-(3/2)(\eta_\Delta/2)}$ Such a function shows a minimum for $\eta_\Delta = \frac{4}{3}$

It then can be concluded from Eq (63) that the shock-layer thickness of a continuously accelerated missile at constant altitude would first take its radiationless value Δ, then reach a minimum for a certain velocity corresponding to $\eta_\Delta = \frac{4}{3}$, and finally return to its radiationless value Δ for the completely self-absorbing case (blackbody) because of very high temperature and velocities

The existence of such a minimum thickness conesponds to a maximum fraction of convected energy lost by radiation; it is the direct analog of similar phenomena in ecology and boiler design [see, for instance, Fig 3 of Ref 30]

In this case we note that the blackbody limit $\eta_\Delta \to \infty$ of the "local temperature approximation" shows the same shock-layer thickness Δ as for the radiationless case This, however, is probably not correct as was shown directly in Sec IIIB: this minimum thickness found out in the case of the horizontal accelerated flight for $n_\Delta = \frac{4}{3}$ is a local minimum only; thinner shock layers will exist for large η_Δ More generally, this "local temperature approximation" does not include the terms leading to the Rosseland approximation when $\eta_\Delta \to \infty$ [3]

It therefore can be concluded that, although "local temperature assumption" results of the type offered in this part for the case of finite optical thickness match the solutions for optically thin gases when $\eta_\Delta \to 0$ and part of those for thick gases when $\eta_\Delta \to \infty$, it is far from being a satisfactory general solution beyond a qualitative indication of the phenomena to be expected [31]

More work, largely of numeiical nature, is needed in this aiea

Conclusion

The detached inviscid radiating shock layer has been analyzed by introducing small perturbations to a simple flow model The role of the optical thickness τ_δ and of the radiation-convection ratio has been illustrated in the approximation of a gray gas Some simple closed-form estimates have been produced which tend to be exact for the asymptotic values of τ_δ and Γ

The shortcomings of this perturbation method for intermediate values of τ_δ and Γ and the fundamental difficulty of matching the radiating inviscid flow with the boundary layer should not be underestimated

References

[1] Riddell, F R and Winkler, H B, "Meteorites and reentry of space systems at meteor velocities," ARS J **32**, 1523 (1962)

[2] Trimpi, R L, Grant, F C, and Cohen, N B, "Aerodynamic and heating problems of advanced reentry vehicles," *Proceedings of the NASA-University Conference on the Science of Technology of Space Exploration*, NASA SP-11 (November 1962), Vol 2, p 235

[3] Goulard, R, "Similarity parameters in radiation gas dynamics,' *Proceedings of the International Symposium on "High Temperature in Aeronautics,'* edited by C Ferrari (Pergamon Press, New York, 1963)

[4] Vaglio-Laurin, R and Ferri, A, "Theoretical investigation of the flow field about blunt nosed bodies in supersonic flight," J Aerospace Sci **25**, 761 (1958)

[5] Gravalos, F G, Edelfelt I H, and Emmons H W "The supersonic flow about a blunt body of revolution for gases at chemical equilibrium," *Ninth International Astronautical Congress* (Springer Verlag, Vienna, 1959), Vol 1 pp 312–332

[6] Hayes, W D and Probstein, R F, *Hypersonic Flow Theory* (Academic Press Inc, New York, 1959), pp 238–240

[7] Goulard, R, "The coupling of radiation and convection in detached shock layers," J Quant Spectry Radiative Transfer **1**, 249–257 (1961)

[8] Freeman, N C, "Non equilibrium flow of an ideal dissociating gas," J Fluid Mech **4**, 407 (1958)

[9] Kennet, H, "Radiation convection interaction around a sphere in hypersonic flow," ARS J **32**, 1616 (1962)

[10] Probstein, R F, "Inviscid flow in the stagnation point region of very blunt nosed bodies at hypersonic flight speeds," Wright Air Dev Center TN 56-395, Armed Services Tech Info Agency AD97273 (September 1956)

[11] Goulard, R and Goulard, M, "One dimensional energy transfer in radiant media," Intern J Heat Mass Transfer **1**, 81–91 (1960)

[12] Kourganoff, V, *Basic Methods in Transfer Problems* (Clarendon Press, Oxford, 1952)

[13] Howe, J T and Viegas, J R, "Solutions of the ionized radiating shock layer, including reabsorption and foreign species effects and stagnation heat transfer," NASA TR R-159, advance copy (1963)

[14] Breene, R G, Naidone M, Riethof, T R, and Zeldin, S, "Radiance of species in high temperature air," Space Sciences Lab, General Electric Missile and Space Vehicle Div Rept R62SD52 (July 1962)

[15] Talbot, L, ' Survey of the shock structure problem" ARS J **32**, 1009 (July 1962)

[16] Teare, J D, Georgiev, S, and Allen, R A, "Radiation from the non equilibrium shock front," Avco-Everett Res Rept 112 (October 1961)

[17] Clarke, J F, "Radiation resisted shock waves," Phys Fluids **5**, 1347–1361 (1962)

[18] Heaslet, M and Baldwin, B, "Predictions on the structure of radiation resisted shock waves,' Phys Fluids **6**, 781–791 (1963)

[19] Yoshikawa, K K and Chapman, D R, "Radiative transfer and absorption behind a hypersonic normal shock wave," NASA TN D-1424 (September 1962)

[20] Guess, A W and Sen, H K, "Radiation effects on Rankine Hugoniot shock conditions, ' *Conference on Extremely High Temperatures* (John Wiley and Sons Inc New York, 1958), pp 111–112

[21] Chen, T and Goulard, R, "The Rankine Hugoniot conditions for radiating air," Purdue Univ, School of Aeronaut and Eng Sci. Rept A&ES 62-14 (January 1963)

[22] Goulard, R, "The transition from blackbody to Rosseland approximations in optically thick flows," Intern J Heat Mass Transfer **6**, 927–930 (1963)

[23] Rosseland, S, *Theoretical Astrophysics* (Oxford University Press, New York, 1936)

[24] Sen, H K and Guess, A W, "Radiation effects in shock wave structure," Phys Rev **108**, 560–564 (1957)

[25] Viskanta, R and Grosh, R J, "Boundary layer in thermal radiation absorbing and emitting media," Intern J Heat Mass Transfer **5**, 795–806 (1962)

[26] Zhigulev, V N, Romishevskyi, Y A, and Vertushkin, V K, "On the role of radiation in modern problems of gas dynamics," Inzh Zh **1**, 60–83 (1961); also AIAA J **1**, 1473–1485 (1963)

[27] Probstein, R F, "Radiation slip," AIAA J **1**, 1202–1204 (1963)

[28] Goulard, R, "Radiation transfer regimes in hypersonic flight," Proc AGARD Combustion and Propulsion Panel Meeting, London, England, April 1963 (to be published)

[29] Lunev, V V and Murzinov, I N, "The effect of radiation on the flow in the neighborhood of the stagnation point of a blunt body," Zh Prikl Mekhan i Tekhn Fiz **1**, 26–30 (March-April 1961)

[30] Adrianov, V N and Shorin, S N, "Radiative transfer in the flow of a radiating medium," Izv Acad Nauk SSSR, Otd Tekhn Nauk (May 1958), transl Purdue Univ, School of Aeronaut and Eng Sci A&ES TT-1 (February 1961); also Atomic Energy Commission TR-3928

[31] Viskanta, R, "Interaction of heat transfer by conduction, convection and radiation in a radiating fluid," Am Soc Mech Engrs Paper 62-WA-189 (1962)

[32] Eschenroeder, A Q, private communication (June 1963)

[33] Scala S M and Sampson, D H, "Heat transfer in hypersonic flow with radiation and chemical reaction," Proc AGARD Combustion and Propulsion Panel Meeting, London, England April 1963 (to be published)